Systemic Analytics

www.sa-journal.org

Syst. Anal. Vol. 3, No. 2 (2025) 118–134.

Paper Type: Original Article

Project Scheduling in Uncertain Environments: An Intuitionistic Fuzzy Approach to solve CPM

Soumyarani Pati[1], **Kshitish Kumar Mohanta**[1,*]

Department of Mathematics Rajendra University, Balangir, 767002, Odisha, India. soumyaranipati12@gmail.com, kshitishkumar.math@gmail.com.

Citation:

Received: 14 December 2024 *Revised: 09 February 2025* *Accepted: 11 March 2025*	Pati, S., & Mohanta, K. K. (2025). Project scheduling in uncertain environments: An intuitionistic fuzzy approach to solve CPM. *Systemic Analytics*, *3*(2), 118-134.

Abstract

The Critical Path Method (CPM) in a fuzzy context is an important tool for dealing with uncertain, perhaps vague, project scheduling. This paper presents two new solution method for the Intuitionistic Fuzzy Critical Path Method (IFCPM) by defining activity durations using Triangular Intuitionistic Fuzzy Numbers (TIFNs). These methods combine forward pass and backward pass calculations, to identify the earliest and latest event times and computes, the total float for each activity. This method improves project scheduling by dealing with uncertainty and hesitancy for activity duration. To demonstrate the practical utility and usefulness of the method, two numerical examples are considered to prove it can identify the critical path in an intuitionistic fuzzy environment. The results demonstrate that the method described here improves the robustness and flexibility of the project scheduling process under uncertainty.

Keywords: Intuitinistic fuzzy set, Project scheduling, Triangular intuitionistic fuzzy number, Critical Path method.

1|Introduction

Fuzzy sets, developed by Zadeh (1965) [1] as a response to limitations in classical sets, provide a means of graded membership that is essential for modeling the concerns of real life that cannot be represented with the binary logic of classical sets and that often contains some level of uncertainty, imprecision, and vagueness. Unlike crisp sets, where an element either completely belongs or does not belong, fuzzy sets allow an element to have a degree of membership between 0 and 1. This capability allows one to consider degrees of memberships in membership functions where distinctions are more subtle as in multi-criteria decision-making [2]. Atanassov (1986) [3] extended fuzzy sets to intuitionistic fuzzy sets (IFSs), which consist of a degree of membership and a degree of non-membership plus a hesitation margin representing a gap of knowledge. IFSs provide a greater degree of flexibility in accommodating uncertainty. Applications of fuzzy sets and IFSs are plentiful and often span a variety of domains such as decision-making, medical diagnosis, pattern recognition, and artificial intelligence [4, 5, 6, 7, 8]. All of these applications highlight the use of fuzzy sets and IFSs as effective means of representing complex and imprecise information, where standard logic cannot provide these types of meaning.

Corresponding Author: kshitishkumar.math@gmail.com
https://doi.org/10.31181/sa32202548

The Critical Path Method (CPM) is a well-known method used for project management that schedules and organizes project activities. It identifies the longest dependent sequence of tasks—that is, the critical path—which defines the project's shortest duration. Traditional CPM is deterministic, meaning activity durations are assumed to be fixed values without any uncertainty associated with them. Obviously, there are many factors in the real world, such as resources, weather, and unforeseen delays, that affect a project duration. Because CPM is unable to treat these uncertainty issues, it limits its use in project situations that are more complex. That's why Fuzzy CPM was created–to provide the time duration uncertainties by using fuzzy set theory to describe the uncertain activities durations. Fuzzy CPM does not assign exact values, but describes activity durations using fuzzy numbers. This type of flex-type approach is a more realistic way to schedule the project while presenting some more un-deterministic time duration. Numerous authors have suggested fuzzy-based methodologies to perform critical path analysis in project networks. Madhuri et al. (2012) [9] developed a linear programming model to establish fuzzy earliest and latest event times using L-L fuzzy numbers via Zadeh's extension principle. Shankar et al. (2010) [10] proposed a metric distance ranking approach to the fuzzy critical path while Khalifa et al. (2021) [11] studied heptagonal fuzzy data to model litigation in scheduling project activities. Other studies investigated other representations of fuzzy numbers like trapezoidal [12], pentagonal, and hexagonal fuzzy numbers [13] with alternative methods to rank and subtract the fuzzy numbers [14, 15]. The literature demonstrates applications of fuzzy critical path methods in hospital emergency departments [16], airport cargo handling [17], and construction scheduling [17]. Other strategies were presented that used fuzzy backward pass modifications to avoid negative values [18] and parametric interval-valued functions, which can be used for network optimization [19]. The research surrounding fuzzy critical path analysis is developing and some of these contributions can be helpful in providing a more solid degree of accuracy with scheduling projects under uncertain conditions. This helps deal with vagueness and imprecision by running a certain range with fuzzy time periods rather than fixed duration information. However, simply defining fuzzy sets defines a membership function, in which fuzzy number runs do not differentiate between uncertainty and hesitation. The fuzzy element doesn't fully serve CPM completely for situations where decision makers are dealing with incomplete or conflicting information.

In order to enhance uncertainty modeling, the Intuitionistic Fuzzy CPM (IFCPM) method is developed. This approach builds upon fuzzy CPM by using intuitionistic fuzzy sets (IFS), which assign both membership and non-membership values in addition to a degree of hesitation. By incorporating this additional parameter, IFCPM facilitates more informed decision-making by holistically capturing the idea of uncertainty. IFCPM provides enhanced flexibility and accuracy for project scheduling and is particularly applicable in uncertain and complex situations when precise information is unavailable. IFCPM allows project managers to address issues of both traditional CPM and fuzzy CPM, and provides a more stable and reliable method for critical path analysis and project management.

The primary purpose of the article is explained below

(1) The Intuitionistic Fuzzy Critical Path Method (IFCPM) is designed by extending the Critical Path Method, utilising Intuitionistic Fuzzy Sets (IFS) to account for uncertainties in the durations and costs of tasks of project management.

(2) IFCPM, using Triangular Intuitionistic fuzzy numbers (TIFNs) adds an additional level of uncertainty as it better describes the vagueness which enhances the reality of project planning and established schedules.

(3) This study proposes two techniques for calculating the critical path based on varying levels of risk associated with a risk factor $\delta \in [0, 1]$.

(4) The possibility mean function is then employed to convert the intuitionistic fuzzy activity into the relevant crisp activity.

The subsequent sections of this article have been organized in a definitive way: Section 2 reviews relevant developments in Intuitionistic Fuzzy Critical Path Method (CPM) and compares them against the overall literature. Section 3 contains the definitions, results, and theorems related to the Intuitionistic Fuzzy Set. Section 4 discusses the development of Intuitionistic Fuzzy CPM by modeling the activity as a Triangular Intuitionistic Fuzzy Number. Section 5 discusses two ways to solve CPM problems and gives the algorithms, and Section

Table 1. Comparison table of the proposed work with the existing work.

Source	Technique	Data type	Purpose
[20]	Decision maker's risk index ranking technique	Interval valid intuitionistic fuzzy numbers	To find the critical path.
[21]	Ranking Technique	Intuitionistic Heptagonal Fuzzy Number	To find the longest path and calculate CPM.
[25]		Fuzzy set	To solve networking problems with uncertain activity durations.
[22]	Ranking Technique	Triangular Intuitionistic fuzzy numbers	To find the critical path applied on intuitionistic fuzzy numbers.
[23]		Triangular Intuitionistic fuzzy numbers	To finding critical path with Intuitionistic fuzzy project network.
[26]	vague parameters	Triangular Intuitionistic fuzzy numbers	To find the critical path in a directed acyclic graph.
[26]		Trapezoidal Intuitionistic fuzzy numbers	To find the critical path in a directed acyclic graph.
[27]	Ranking Method and Graphical Method	Trapezoidal Intuitionistic fuzzy numbers	Application of CPM
[9]	α-cuts	L-L fuzzy numbers	To calculate fuzzy earliest and fuzzy latest events.
[10]	metric distance ranking method	Trapezoidal Intuitionistic fuzzy numbers	To find the critical path.
[28]	Ranking technique	Fuzzy numbers	To find the fuzzy critical path
[18]	Fuzzy backward pass technique	Negative fuzzy numbers	To obtain the fuzzy event latest,fuzzy activity latest start and fuzzy activity latest finish times
[12]		Trapezoidal Intuitionistic fuzzy numbers	to find latest fuzzy finish time
[19]		parametric interval valued function	To solve the parametric network problem.
[15]	Ranking method	octagonal fuzzy number	To determine the critical path
This work	Possibility mean function	Triangular intuitionistic fuzzy numbers	To find the fuzzy critical path.

6 include two numerical examples to demonstrate the applied nature of the methodology. Finally, Section 7 concludes with comments and thoughts about the findings as well as possible directions for future research.

2|Litreture Review

The intuitionistic fuzzy critical path method (CPM) enhances traditional project scheduling techniques by incorporating intuitionistic fuzzy numbers (IFNs), which consider membership, non-membership, and hesitation degrees. This approach provides a more flexible and realistic representation of uncertain activity durations. Several researchers have contributed to this field by introducing novel ranking techniques and fuzzy number representations. Kiruthiga and Hemalatha (2019) [20] applied interval-valued intuitionistic fuzzy numbers (IVIFNs) to determine the critical path, incorporating decision-maker risk indices for improved decision-making. Similarly, Rameshan and Dinagar (2020) [21] proposed an alternative approach using Intuitionistic Heptagonal Fuzzy Numbers (IHFNs) and an innovative defuzzification method, proving effective in identifying the longest path. Other studies, such as Porchelvi and Sudha (2015) [22] and Sudha et al. (2017) [23], utilized triangular intuitionistic fuzzy numbers (TIFNs) for ranking and optimization, demonstrating improved accuracy in handling vague project parameters. The literature on project scheduling extensively covers fuzzy and intuitionistic fuzzy approaches, with applications in various sectors. Madhuri et al. (2012) [9] and Shankar et al. (2010) [10] explored linear programming models and ranking methods for fuzzy CPM, while Khalifa et al. (2021) [11] introduced heptagonal fuzzy data to resolve scheduling conflicts. Additionally, Ganesh and Shobana (2020) [13] proposed trisectional and pentasectional fuzzification techniques for optimizing interval-based CPM problems. Applications of fuzzy CPM extend across industries, including hospital emergency departments [16], airport cargo handling [24], and defense projects. However, research on intuitionistic fuzzy CPM in the defense industry remains limited. Most studies focus on conventional CPM and PERT methodologies for project scheduling, highlighting the need for further research in applying intuitionistic fuzzy approaches to defense-related projects for more precise scheduling and risk assessment.

3|Preliminary Concepts

In this section, we consider to provide the instruments that made this idea come to our mind. To start, we suppose that (Ω, F, Q) is the probability measure space where Ω is a sample space, F is an event space and Q is a probability function and we work in this space. In the following, we will define the Exchange and Lookback options and show the price equation. To this end, we consider the following two risky assets:

Definition 1 ([1]). *Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$, where $\mu_A(x) : X \to [0,1]$ is the membership function of the fuzzy set A. Fuzzy set is a collection of objects with graded membership i.e. having degrees of membership.*

Definition 2 ([3]). *Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{\langle x, \mu_A(x), v_A(x)\rangle : x \in X\}$, where the functions $\mu_A(x), v_A(x) : X \to [0,1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A , which is a subset of X, and for every element $x \in X, 0 \le \mu_A(x) + v_A(x) \le 1$.*

Definition 3 (**Intuitionistic Fuzzy Number**). *The set of real numbers defined on the fuzzy set is called intuitionistic fuzzy number if $\tilde{A} = (x, \mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x)) : x \in R$*

(1) *There exist a real number $x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$ and $\gamma_{\tilde{A}}(x_0) = 0$.*

(2) *$\mu_{\tilde{A}}$ of $\tilde{A}$ is fuzzy convex and upper semi- continuous.*

(3) *$\gamma_{\tilde{A}}$ of A is fuzzy concave and lower semi -continuous.*

Definition 4. ***Trapezoidal Intuitionistic Fuzzy Number(TRIFN)***
An intuitionistic fuzzy number A=<a,b,c,d><a',b',c',d'> is said to be trapezoidal intuitionistic fuzzy number if its membership function and non-membership function are given by

$$\mu_A = \begin{cases} \frac{(x-a)}{(b-a)}, a \le x \le b \\ 1, b \le x \le c \\ \frac{(x-d)}{(c-d)}, c \le x \le d \\ 0, otherwise \end{cases} \quad and\ \nu_A(x) = \begin{cases} \frac{(b'-x)}{(b'-a')}, a' \le x \le b' \\ 0, b' \le x \le c' \\ \frac{(x-c')}{(d'-c')}, c' \le x \le d' \\ 1, otherwise \end{cases}$$

Definition 5 ([6]). *Let $\hat{A} = \langle a^L, a^M, a^U; \phi_a, \psi_a\rangle$ be a triangular intuitionistic fuzzy number (TIFN).*

(1) *The possibility mean of truth membership function for $\hat{A}$ is defined as follows:*

$$M(\hat{A}_\alpha) = \frac{m_L(\hat{A}_\alpha) + m_U(\hat{A}_\alpha)}{2}$$

where

$$m_L(\hat{A}_\alpha) = 2\int_0^{\phi_a} Pos[\hat{A} \le \hat{A}_L(\alpha)]\hat{A}_L(\alpha)d\alpha = \frac{a^L + 2a^M}{3}\phi_a^2$$
$$and\ m_U(\hat{A}_\alpha) = 2\int_0^{\phi_a} Pos[\hat{A} \le \hat{A}_U(\alpha)]\hat{A}_U(\alpha)d\alpha = \frac{a^U + 2a^M}{3}\phi_a^2$$

(2) *The possibility mean of falsity membership function for $\hat{A}$ is defined as follows:*

$$M(\hat{A}_\gamma) = \frac{m_L(\hat{A}_\gamma) + m_U(\hat{A}_\gamma)}{2}$$

where

$$\begin{aligned} m_L(\hat{A}_\beta) &= 2\int_{\psi_a}^{1} Pos[\hat{A} \geq \hat{A}_L(\beta)]\hat{A}_L(\beta)d\beta \\ &= (a^M - \psi_a a^L)(1+\psi_a) - \frac{2(a^M - a^L)}{3}(1+\psi_a+\psi_a^2) \\ \textit{and } m_U(\hat{A}_\beta) &= 2\int_{\psi_a}^{1} Pos[\hat{A} \geq \hat{A}_U(\beta)]\hat{A}_U(\beta)d\beta \\ &= (a^M - \psi_a a^U)(1+\psi_a) + \frac{2(a^U - a^M)}{3}(1+\psi_a+\psi_a^2) \end{aligned}$$

Definition 6 ([6]). *Let $\hat{A} = \langle a^L, a^M, a^U; \phi_a, \psi_a\rangle$ be a triangular intuitionistic fuzzy number (TIFN)s.The weighted possibility means of truth and falsity membership function can be defined as*

$$\widetilde{\mathcal{R}}(\widehat{A}) = \delta\mathcal{M}(\widehat{A}_\alpha) + (1-\delta)\mathcal{M}(\widehat{A}_\beta)$$

and $\delta \in [0,1]$ reflects the attitude of the decision maker towards the risk.

(1) $\delta \in [0, 0.5)$ *shows that the expert is a risk taker who prefers uncertainty.*

(2) $\delta = 0.5$ *shows that the expert's decision on the parameter selection is neutral.*

(3) $\delta \in (0.5, 1]$*shows that the expert's sensitivity to taking risks while making a decision.*

That implies

$$\widetilde{\mathcal{R}}(\widehat{A}) = \delta(\frac{a^L + 4a^M + a^U}{6})\phi_a^2 + (1-\delta)\Big(\frac{2[a^L + a^M + a^U] - [a^L - 2a^M + a^U]\psi_a - [a^L + 4a^M + a^U]\psi_a^2}{6}\Big) \tag{1}$$

Definition 7. *Suppose $\widehat{A}_1 and \widehat{A}_2$ be two TIFNs,then the following are satisfy*

(1) $\widehat{A}_1 \leq \widehat{A}_2$ *if and only if* $\widetilde{\mathcal{R}}(\widehat{A}_1) \leq \widetilde{\mathcal{R}}(\widehat{A}_2)$

(2) $\widehat{A}_1 < \widehat{A}_2$ *if and only if* $\widetilde{\mathcal{R}}(\widehat{A}_1) < \widetilde{\mathcal{R}}(\widehat{A}_2)$

(3) $A_1 A_2$ *if and only if* $\widetilde{\mathcal{R}}(\widehat{A}_1) = \widetilde{\mathcal{R}}(\widehat{A}_2)$

Theorem 1 ([6]). *Consider $\widetilde{A} = \langle a_i^L, a_i^M, a_i^U, \phi_{a_i}, \psi_{a_i}\rangle$ be n TIFNs and $\lambda_i i \in R$ .Then the possibility mean of the aggregation of the following expression can be defined as*

$$\begin{aligned} \widetilde{\mathcal{R}}(\sum_{i=1}^{n} \lambda_i \widehat{A}_i) = \sum_{i=1}^{n} \Bigg[&\delta(\frac{a_i^L + 4a_i^M + a_i^U}{6})(\bigwedge_{i=1}^{n} \phi_{a_i})^2 \\ &+ (1-\delta)\Big(\frac{2[a_i^L + a_i^M + a_i^U] - [a_i^L - 2a_i^M + a_i^U](\bigvee_{i=1}^{n} \psi_{a_i}) - [a_i^L + 4a_i^M + a_i^U](\bigvee_{i=1}^{n} \psi_{a_i})^2}{6}\Big)\Bigg]\lambda_i \end{aligned} \tag{2}$$

4|Development of Intuitionistic Fuzzy CPM

Table 2 shows the essential notations in the IFCPM which generalizes the classical approach to project scheduling by adding uncertainty through intuitionistic fuzzy sets.

Table 2. Notations for Intuitionistic Fuzzy CPM.

Notation	**Description**
N	The set of all nodes in a project network.
T_{ij}	The intuitionistic fuzzy activity time from the ith node to the jth node.
EST_j	The intuitionistic fuzzy earliest starting time of node j.
LFT_j	The intuitionistic fuzzy latest finishing time of node j.
TS_{ij}	The intuitionistic fuzzy total float of the activity between nodes i and j.
$S(j)$	The set of all successor activities of node j.
$NS(j)$	The set of all nodes connected to all successor activities of node j, i.e., $NS(j) = \{k \mid A_{jk} \in S(j), k \in N\}$.
$F(j)$	The set of all predecessor activities of node j.
$NP(j)$	The set of all nodes connected to all predecessor activities of node j, i.e., $NP(j) = \{i \mid A_{ij} \in F(j), i \in N\}$.
P_i	The ith path in the project network.
P	The set of all paths in a project network.
$IFCP(P_k)$	The intuitionistic fuzzy completion time of path P_k in a project network.

An abstract diagram is given in Figure 1, which shows that the Intuitionistic Fuzzy CPM/PERT for better understand. The activities are considered as triangular intuitionistic fuzzy number.

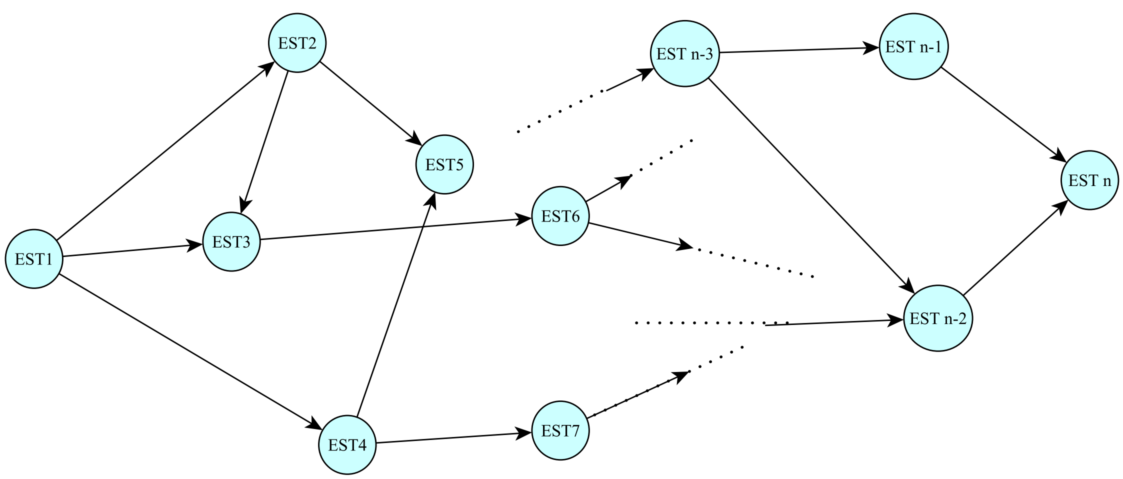

Figure 1. Conceptual Diagram for CPM.

5|Solutions Techniques for Intuitionistic CPM/PERT

There are two technique proposed to solve the intuitionistic fuzzy CPM/PERT. These techniques are used for finding the CPM/PERT problem under intuitionistic fuzzy environment.

5.1|Technique1

Forward Pass Calculation

(i) choose the risk factor $\delta \in [0, 1]$.

(ii) calculate the possibility activity duration by using possibility mean function.

(iii) The computation begins from the start node and move towards the end node. For easiness, the forward pass computation starts by assuming the earliest occurrence time of zero for the initial project event.

(iv) Earliest starting time of activity (i, j) is the earliest event time of the tail end event i.e. $(Es)_{ij} = E_i$.

(v) Earliest finish time of activity (i, j) is the earliest starting time + the activity time i.e. $(Ef)_{ij} = (Es)_{ij} + T_{ij}$ or $(Ef)_{ij} = E_i + Tij$.

(vi) Earliest event time for event j is the maximum of the earliest finish times of all activities ending in to that event i.e. $E_j = max[(Ef)_{ij}$ for all immediate predecessor of $(i, j)]$ or $E_j = max[E_i + T_{ij}]$.

Backward Pass Calculation

(i) choose the risk factor $\delta \in [0, 1]$.

(ii) calculate the possibility activity duration by using possibility mean function.

(iii) For ending event assume $E = L$ .Remember that all E's have been computed by forward pass computations.

(iv) Latest finish time for activity (i, j) is equal to the latest event time of event j i.e. $(Lf)_{ij} = L_j$.

(v) Latest starting time of activity (i, j) = the latest completion time of $(i, j)-$ the activity time or $(Ls)_{ij} = (Lf)_{ij} - T_{ij}$ or $(Ls)_{ij} = L_j - T_{ij}$.

(vi) Latest event time for event i is the minimum of the latest start time of all activities originating from that event i.e. $L_i = min[(Ls)_{ij}$ for all immediate successor of $(i, j)] = \min\ [(Lf)_{ij} - T_{ij}] = \min\ [L_j - T_{ij}]$.

5.2|Solution Technique 2

Forward Pass Calculation

(1) Construct the fuzzy project network using intuitionistic fuzzy numbers as activity durations and assign event numbers accordingly.

(2) Represent each intuitionistic fuzzy activity duration in parametric form.

(3) Assume that the intuitionistic fuzzy earliest starting time of the initial event is zero, i.e.,

$$EST_1 = \langle 0, 0, 0; 1, 0\rangle$$

(4) Compute the earliest start time for each event using:

$$EST_j = \max\{EST_i \oplus T_{ij} \mid i \in NP(j), j \neq 1\}, \quad j = 2, 3, \ldots, n$$

where $NP(j)$ represents the set of all predecessor nodes of event j.

(5) Using the ranking function equation (1), choose the maximum value of EST_j.

(6) Compute the intuitionistic fuzzy earliest finish time for each activity using:

$$EFT_j = EST_j \oplus T_{ij}$$

Backward Pass Calculation

(1) Assume $LFT_n = EST_n$, since the intuitionistic fuzzy latest finish time of all the end activities is taken as the earliest completion time of the project network. Compute LFT_j for $j = n-1, n-2, \dots, 2, 1$ using:

$$LFT_j = \min\{LFT_k - T_{jk} \mid k \in NS(j)\}$$

where $NS(j)$ is the set of all successor nodes of j.

(2) Using the ranking function equation (1), choose the minimum value of LFT_j.

(3) The intuitionistic fuzzy latest start time for each activity is given by:

$$LST_{ij} = LFT_j - T_{ij}$$

(4) Calculate the total float for each activity:

$$TF_{ij} = \min(LFT_j - (EST_i \oplus T_{ij}))$$

(5) An activity is said to be a **critical activity** if and only if its total float is zero, i.e.,

$$TF_{ij} = 0$$

The **critical path** is the longest path from the initial event to the terminal event in the project network, having the maximum duration. All activities in a critical path are called **critical activities**.

5.3|Algorithms

Algorithm 1 Algorithm for Technique 1

Choose the risk factor $\delta \in [0, 1]$.
Calculate the possibility activity duration using the possibility mean function.
Initialize the earliest occurrence time $E_0 = 0$ for the start node.
for each activity (i, j) moving from start to end **do**
    Set earliest starting time: $(Es)_{ij} = E_i$.
    Compute earliest finish time: $(Ef)_{ij} = (Es)_{ij} + T_{ij}$ or $(Ef)_{ij} = E_i + T_{ij}$.
    Update earliest event time for event j:

$$E_j = \max\{(Ef)_{ij} \mid \text{for all immediate predecessors } (i, j)\}.$$

end for
Set latest event time L for the end node: $L = E$ (obtained from the forward pass).
for each activity (i, j) moving from end to start **do**
    Set latest finish time: $(Lf)_{ij} = L_j$.
    Compute latest starting time: $(Ls)_{ij} = (Lf)_{ij} - T_{ij}$ or $(Ls)_{ij} = L_j - T_{ij}$.
    Update latest event time for event i:

$$L_i = \min\{(Ls)_{ij} \mid \text{for all immediate successors } (i, j)\}.$$

end for

The algorithm for solution technique 1 and solution technique 2 are given in Algorithm 1 and Algorithm 2.

6|Numerical Examples

A relevant project case can validate and illustrate the computational process of intuitionistic fuzzy critical path analysis. The project activities are denoted by intuitionistic fuzzy number representations, with sequencing done by performing the forward and backward pass, along with fuzzy arithmetic and ranking functions. This is a plausible approach to uncertainty for a project schedule as it offers a more flexible and realistic specification of project constraints than using deterministic processes.

Algorithm 2 Algorithm for Technique 2

Identify activities in a project.
Establish precedence relationships of all activities.
Estimate the intuitionistic fuzzy activity time for each activity.
Construct the project network.
Set the initial node to be zero for starting, i.e., $EST_1 = \langle 0, 0, 0; 1, 0\rangle$.
for $j = 2$ to n **do**
    Compute EST_j using:

$$EST_j = \max\{EST_i \oplus T_{ij} \mid i \in NP(j), i \neq 1, j \in N\}$$

end for
Choose the maximum value of EST_j using the possibility function, given in equation (1).
Set $LFT_n = EST_n$.
for $j = n - 1$ to 1 **do**
    Compute LFT_j using:

$$LFT_j = \min\{LFT_k - T_{ik} \mid k \in NS(j), j \neq n, j \in N\}$$

end for
Compute total slack TS_{ij} for each activity:

$$TS_{ij} = LFT_j - (EST_i \oplus T_{ij}), \quad 1 \leq i < j \leq n,\ i, j \in N$$

Choose the minimum value of LFT_j using the possibility function, given in equation (1).
Find all possible paths and calculate $IFCP(P_k)$ using:

$$IFCPM(P_k) = \sum_{1 \leq i < j \leq n} TS_{ij}, \quad P_k \in P$$

Determine the intuitionistic fuzzy critical path using:

$$IFCPM(P_c) = \min\{IFCPM(P_i) \mid P_i \in P\}$$

The path P_c is the intuitionistic fuzzy critical path.
Compute the possibility mean of project completion at the scheduled time.

Example 1. *A project consisting of nine distinct activities, each specified by a particular activity dependence and an intuitionistic fuzzy activity duration represented as Triangular Intuitionistic Fuzzy Numbers (TIFNs) shown in Table 3.*

Table 3. The fuzzy activity time for each activity in the project network.

Activity	*Preceding Activity*	*Intuitionistic Fuzzy Activity time* T_{ij}
$A(1 \to 2)$	-	$\langle 1, 6, 8, ; 0.5, 0.4\rangle$
$B(2 \to 3)$	A	$\langle 1, 7, 8, ; 0.4, 0.3\rangle$
$C(2 \to 4)$	A	$\langle 2, 5, 7, ; 0.6, 0.1\rangle$
$D(2 \to 5)$	A	$\langle 4, 6, 7, ; 0.8, 0.2\rangle$
$E(2 \to 7)$	A	$\langle 5, 7, 9, ; 0.4, 0.5\rangle$
$F(3 \to 6)$	B	$\langle 0, 0, 0, ; 1, 0\rangle$
$G(3 \to 7)$	B	$\langle 4, 5, 9, ; 0.7, 0.2\rangle$
$H(4 \to 5)$	C	$\langle 0, 0, 0, ; 1, 0\rangle$
$I(5 \to 6)$	D	$\langle 2, 6, 9, ; 0.5, 0.3\rangle$
$J(6 \to 8)$	*B and I*	$\langle 1, 8, 9, ; 0.6, 0.3\rangle$
$K(7 \to 8)$	*E and G*	$\langle 3, 6, 8, ; 0.5, 0.2\rangle$
$L(8 \to 9)$	*J and K*	$\langle 2, 7, 8, ; 0.3, 0.5\rangle$

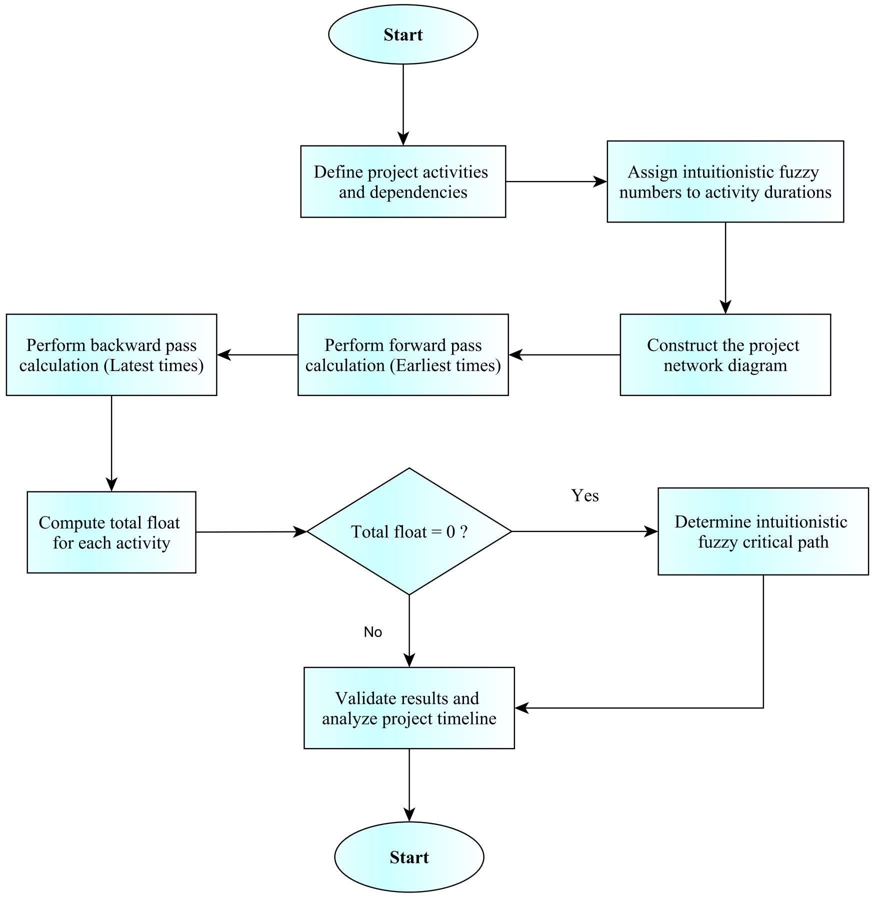

Figure 2. Flow chart for intuitionistic fuzzy CPM.

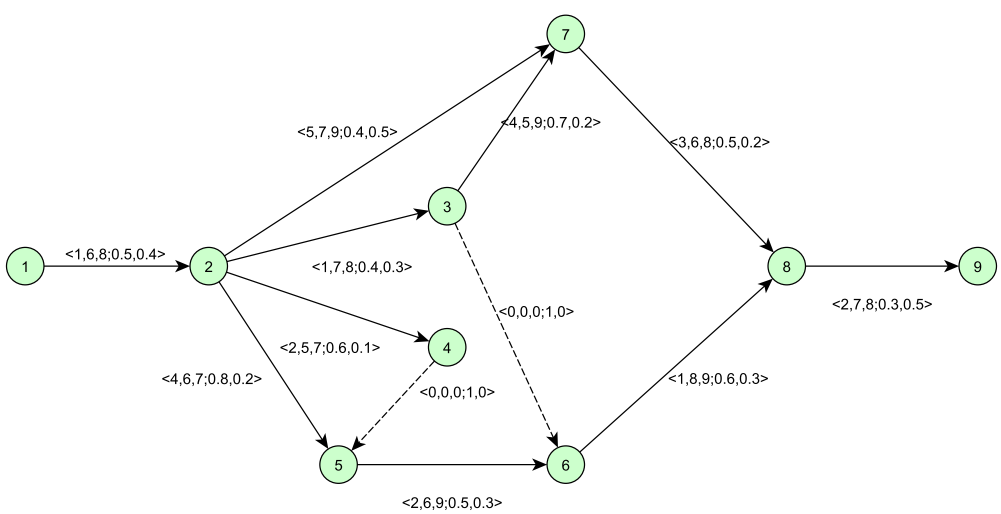

Figure 3. Network diagram of Example 1.

In this project network, the activity durations are expressed as Triangular Intuitionistic Fuzzy Numbers (TIFNs) in order to address uncertainty and hesitation. With Technique 1, the intuitionistic fuzzy durations are defuzzified

TABLE 4. The crisp values of the Intuitionistic fuzzy activity time with different risk factor $\delta \in [0, 1]$

	$\delta=0$	$\delta=0.1$	$\delta=0.2$	$\delta=0.3$	$\delta=0.4$	$\delta=0.5$	$\delta=0.6$	$\delta=0.7$	$\delta=0.8$	$\delta=0.9$	$\delta=1$
T_{12}	4.32	4.0255	3.731	3.4365	3.142	2.8475	2.553	2.2585	1.964	1.6695	1.375
T_{23}	5.0283	4.6242	4.22	3.8158	3.4117	3.0075	2.6033	2.1992	1.795	1.3908	0.9867
T_{24}	4.635	4.3455	4.056	3.7665	3.477	3.1875	2.898	2.6085	2.319	2.0295	1.74
T_{25}	5.4667	5.2933	5.12	4.9467	4.7733	4.6	4.4267	4.2533	4.08	3.9067	3.7333
T_{27}	5.25	4.837	4.424	4.011	3.598	3.185	2.772	2.359	1.946	1.533	1.12
T_{37}	5.68	5.3815	5.083	4.7845	4.486	4.1875	3.889	3.5905	3.292	2.9935	2.695
T_{36}	0	0	0	0	0	0	0	0	0	0	0
T_{45}	0	0	0	0	0	0	0	0	0	0	0
T_{56}	5.1917	4.8183	4.445	4.0717	3.6983	3.325	2.9517	2.5783	2.205	1.8317	1.4583
T_{68}	5.67	5.355	5.04	4.725	4.41	4.095	3.78	3.465	3.15	2.835	2.52
T_{78}	5.4667	5.0658	4.665	4.2642	3.8633	3.4625	3.0617	2.6608	2.26	1.8592	1.4583
T_{89}	4.4167	4.032	3.6473	3.2627	2.878	2.4933	2.1087	1.724	1.3393	0.9547	0.57

TABLE 5. Critical path analysis of the Example 1

		$\delta=0$	$\delta=0.1$	$\delta=0.2$	$\delta=0.3$	$\delta=0.4$	$\delta=0.5$	$\delta=0.6$	$\delta=0.7$	$\delta=0.8$	$\delta=0.9$	$\delta=1$
EST5	M1	9.12	8.4913	7.8627	7.234	6.6053	5.9767	5.348	4.7193	4.0907	3.462	2.8333
	M2	8.28	7.6173	6.9547	6.292	5.6293	4.9667	4.304	3.6413	2.9787	2.316	1.6533
EST6	M1	13.92	12.9572	11.9943	11.0315	10.0687	9.1058	8.143	7.1802	6.2173	5.2545	4.2917
	M2	9	8.3917	7.7853	7.175	6.5667	5.9583	5.35	4.7417	4.1333	3.525	2.9167
EST7	M1	9.125	8.4125	7.7	6.9875	6.275	5.5625	4.85	4.1375	3.425	2.7125	2
	M2	13.92	12.8027	11.6853	10.568	9.4507	8.333	7.216	6.0987	4.9813	3.864	2.7467
EST8	M1	19.2	17.8842	16.5683	15.2525	13.9367	12.6208	11.305	9.9892	8.6733	7.3575	6.0417
	M2	18.72	17.216	15.712	14.208	12.704	11.2	9.696	8.192	6.688	5.184	3.68
EST9	M1	21.625	19.737	17.849	15.961	14.073	12.185	10.297	8.409	6.521	4.633	2.745
LFT3	M1	12.4583	11.367	10.2757	9.1843	8.093	7.0017	5.9103	4.819	3.7277	2.6363	1.545
	M2	8.5417	7.803	7.0643	6.3257	5.587	4.8483	4.1097	3.371	2.6323	1.8937	1.155
LFT2	M1	7.6667	7.002	6.3373	5.6727	5.008	4.3433	3.6787	3.014	2.3493	1.6847	1.02
	M2	3.875	3.537	3.199	2.861	2.523	2.185	1.847	1.509	1.171	0.833	0.495

into crisp values based on a risk parameter $\delta \in [0, 1]$, that is, δ controls the degree of optimism or pessimism with respect to uncertainty.

Using the defuzzified times in Table 4, we can make two key observations: First, as the risk factor δ increases from 0 (very optimistic) to 1 (very pessimistic), the crisp activity times become shorter. This seems to result from a tendency for caution or increasing risk aversion results in shorter effective durations.

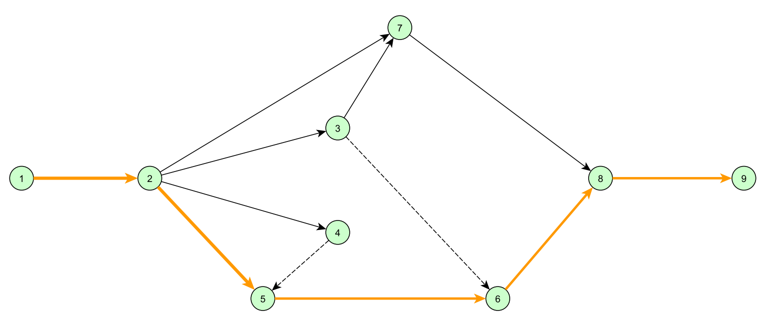

Figure 4. Critical Path for Example 1.

Using Technique 2, the critical path analysis is carried out for each value of $\delta \in [0, 1]$. The critical path, for any value of δ, also is the same through the whole interval [0,1]. Thus, the Critical path is highlated in Figure 4 i.e.,

$$1 \to 2 \to 5 \to 6 \to 8 \to 9.$$

The critical path is identified in the network.

Table 6 and Figure 5 show the variation in activity durations with respect to changing risk factors (δ) using two proposed techniques for the IFCPM. As the risk factor changes from 0 to 1, both techniques demonstrate a similar downward trajectory of project duration, as pessimism decreases, and spreads diverse views of more optimism. Technique 1 declines from a duration of 25.0651 at $\delta = 0$ to 9.6566 at $\delta = 1$, while Technique 2 declines from 21.625 to 2.745 appropriate to its higher course of adjustment from state to state. Figure 5) clearly demonstrates this relationship, while also bringing attention to Technique 2 as the clearly more adaptable option. The results suggest the seriousness of appending the risk profile to a project schedule and the idea that Technique 2 provides more efficient schedules under greater uncertainties (less optimism), and therefore Technique 2 may be the more appropriate option in both dynamic and complex project environments.

Table 6. Activity Duration with respect to different risk factor.

Risk factor (δ)	Technique 1	Technique 2
0	25.0651	21.625
0.1	23.5241	19.737
0.2	21.9833	17.849
0.3	20.4426	15.961
0.4	18.9061	14.073
0.5	17.3608	12.185
0.6	15.8201	10.297
0.7	14.2783	8.409
0.8	12.7383	6.521
0.9	11.1976	4.633
1	9.6566	2.745

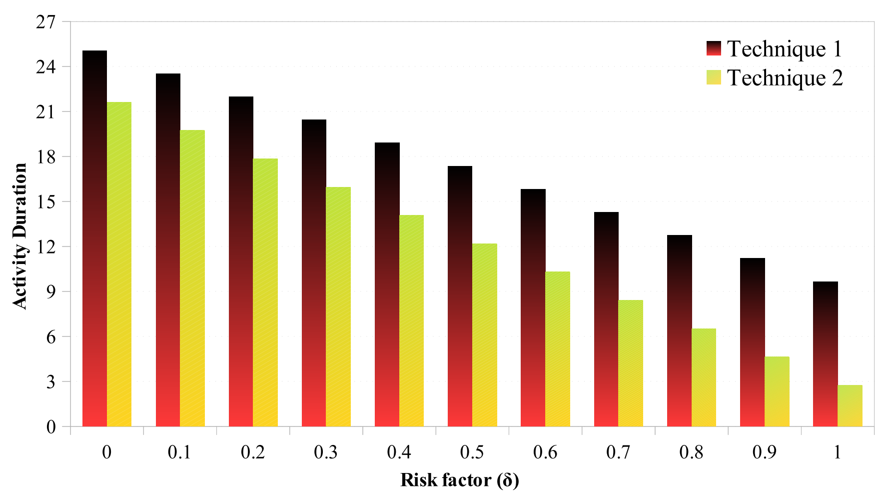

Figure 5. Comaparision of Activity duration using Technique 1 and Technique 2.

Example 2. *A project consisting of eight distinct activities, each specified by a particular activity dependence and an intuitionistic fuzzy activity duration represented as Triangular Intuitionistic Fuzzy Numbers (TIFNs) shown in Table 7.*

Solution: The network diagram consists of set of node $N = \{1, 2, 3, 4, 5, 6, 7, 8\}$, the intuitionistic fuzzy activity time for each activity are shown in Figure 6.

In this project network, the activity durations are expressed as Triangular Intuitionistic Fuzzy Numbers (TIFNs) in order to address uncertainty and hesitation. With Technique 1, the intuitionistic fuzzy durations are defuzzified into crisp values based on a risk parameter $\delta \in [0, 1]$, that is, δ controls the degree of optimism or pessimism with respect to uncertainty.

Table 7. The fuzzy activity time for each activity in the project network.

Activity	*Intuitionistic Fuzzy Activity time* T_{ij}
$1 \to 2$	$\langle 1, 3, 5; 0.8, 0.2 \rangle$
$1 \to 3$	$\langle 2, 4, 6; 0.7, 0.1 \rangle$
$2 \to 3$	$\langle 3, 5, 7; 0.4, 0.5 \rangle$
$2 \to 4$	$\langle 4, 6, 8; 0.1, 0.8 \rangle$
$2 \to 5$	$\langle 5, 7, 9; 0.4, 0.3 \rangle$
$3 \to 6$	$\langle 6, 8, 9; 0.6, 0.2 \rangle$
$4 \to 5$	$\langle 0, 0, 0; 1, 0 \rangle$
$4 \to 7$	$\langle 2, 6, 8; 0.1, 0.7 \rangle$
$5 \to 7$	$\langle 3, 7, 9; 0.5, 0.3 \rangle$
$6 \to 8$	$\langle 4, 7, 8; 0.3, 0.2 \rangle$
$7 \to 8$	$\langle 5, 6, 9; 0.6, 0.3 \rangle$

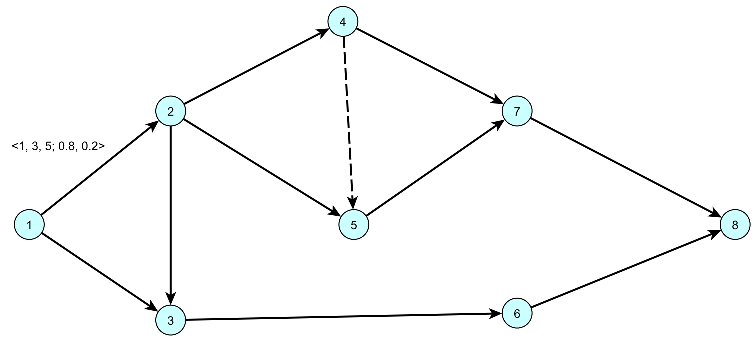

Figure 6. Network diagram for Example 2.

Table 8. The crisp values of the Intuitionistic fuzzy activity time with different risk factor ∈ [0, 1].

	$\delta = 0$	$\delta = 0.1$	$\delta = 0.2$	$\delta = 0.3$	$\delta = 0.4$	$\delta = 0.5$	$\delta = 0.6$	$\delta = 0.7$	$\delta = 0.8$	$\delta = 0.9$	$\delta = 1$
12	2.88	2.784	2.688	2.592	2.496	2.4	2.304	2.208	2.112	2.016	1.92
13	3.96	3.76	3.56	3.36	3.16	2.96	2.76	2.56	2.36	2.16	1.96
23	3.75	3.455	3.16	2.865	2.57	2.275	1.98	1.685	1.39	1.095	0.8
24	2.16	1.95	1.74	1.53	1.32	1.11	0.9	0.69	0.48	0.27	0.06
25	6.37	5.845	5.32	4.795	4.27	3.745	3.22	2.695	2.17	1.675	1.12
36	7.3867	6.93	6.4733	6.0167	5.56	5.1033	4.6467	4.19	3.7333	3.2767	2.82
45	0	0	0	0	0	0	0	0	0	0	0
47	2.79	2.5167	2.2433	1.97	1.6967	1.4233	1.15	0.8767	0.6033	0.33	0.0567
57	5.8333	5.4167	5	4.5833	4.1667	3.75	3.333	2.9167	2.5	2.0833	1.6667
68	6.1333	5.58	5.0267	4.4733	3.92	3.3667	2.8133	2.26	1.7067	1.1533	0.6
78	5.9967	5.625	5.2533	4.8817	435100	4.1383	3.7667	3.395	3.0233	2.6517	2.28

Using the defuzzified times in Table 8, we can make two key observations: First, as the risk factor δ increases from 0 (very optimistic) to 1 (very pessimistic), the crisp activity times become shorter. This seems to result from a tendency for caution or increasing risk aversion results in shorter effective durations.

Using Technique 2, the critical path analysis is carried out for each value of $\delta \in [0, 1]$. The critical path, for any , also is the same through the whole interval [0,1]. Thus, the Critical path is highlated in Figure 7 i.e.,

$$1 \to 2 \to 5 \to 7 \to 9.$$

The critical path is identified in the network.

The results obtained from both methods for solving the IFCPM indicate that the activity duration has consistently decreased across both methods as the risk factor (δ) shifted between 0 and 1 as shown in Table 10. For instance,

Table 9. Critical path analysis of the Example 2.

		$\delta = 0$	$\delta = 0.1$	$\delta = 0.2$	$\delta = 0.3$	$\delta = 0.4$	$\delta = 0.5$	$\delta = 0.6$	$\delta = 0.7$	$\delta = 0.8$	$\delta = 0.9$	$\delta = 1$
E3	M1	3.96	3.76	3.56	3.36	3.16	2.96	2.76	2.56	2.36	2.16	1.96
	M2	6	5.528	5.056	4.584	4.112	3.64	3.168	2.696	2.224	1.752	1.28
E5	M1	9.1	8.35	7.6	6.85	6.1	5.35	4.6	3.85	3.1	2.35	1.6
	M2	3.24	3.92	2.61	2.29	1.98	1.665	1.35	1.035	0.72	0.405	0.9
E7	M1	5.2133	4.7067	4.2	3.6933	3.1867	2.68	2.1733	1.6667	1.16	0.6533	0.1467
	M2	14.9333	13.7067	12.48	11.2533	10.0267	8.8	7.5733	6.3467	5.12	3.8933	2.6667
E8	M1	16.625	15.165	13.705	12.245	10.785	9.325	7.865	6.405	4.945	3.485	2.025
	M2	20.93	19.205	17.48	15.755	14.03	12.305	10.58	8.855	7.13	5.405	3.68
L4	M1	9.1	8.35	7.6	6.85	6.1	5.35	4.06	3.85	3.1	2.35	1.6
	M2	5.61	5.06	4.51	3.96	3.41	2.86	2.31	1.76	1.21	0.66	0.11
	M1	2.73	2.505	2.28	2.055	1.83	1.605	1.38	1.155	0.93	0.705	0.48
L2	M2	1.44	1.3	1.16	1.02	0.88	0.74	0.6	0.46	0.32	0.18	0.04
	M3	2.875	2.619	2.363	2.107	1.851	1.595	1.339	1.083	0.827	0.571	0.315
L1	M1	0.36	0.325	0.29	0.255	0.22	0.185	0.15	0.115	0.08	0.045	0.0106
	M2	3.64	3.312	2.984	2.656	2.328	2	1.672	1.344	1.016	0.688	0.36

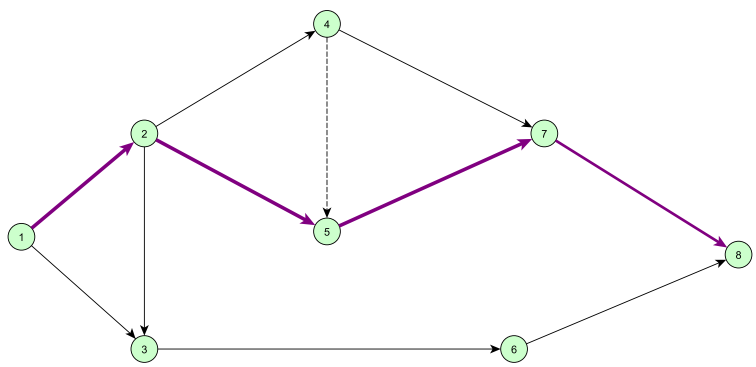

Figure 7. The highlighted critical path for Example 2.

with Technique 1, the first duration output at $\delta = 0$ was 21.08 units while the lowest was 6.9867 units at $\delta = 1$. Technique 2 on the other hand, provided a first duration of 20.93 units at $\delta = 0$ and reduced the activity duration significantly down to 3.68 units at $\delta = 1$. This trend corresponds to the risk averse nature of the solution that was produced. Higher risk acceptance meant higher project durations, whereas low risk acceptance had optimistic durations. The plot used for comparison in Figure 8 showed that based on all risk levels tested in this research, Technique 2 always produced durations that were lower than technique 1. This indicates that the technique is more sensitive to the various risk levels that were inputted into the models. The spread seen between the two methods also suggests that Technique 2 could have been the more appropriate method to use if the work was expected to be a lot more aggressive with higher uncertainty. Thus, the results confirm the usability and potential of the IFCPM framework as it relates to incorporating uncertainty using a risk factor, as well as show how various computational strategies to calculating risk can affect the final project scheduling results.

7|Conclusion and Future Directions

The suggested techniques for addressing the Intuitionistic Fuzzy Critical Path Method (IFCPM) effectively tackles the uncertainty and hesitation in activity durations using Triangular Intuitionistic Fuzzy Numbers (TIFNs). Conducting the forward and backward pass calculations provides a structured method to determine the earliest start time (EST) and latest finish time (LFT), and allows one to correctly calculate the total float for each activity. The developed algorithms demonstrate that the IFCPM methods can ultimately be used in real-life project scheduling. The sensitivity of the critical path was investigated by adjusting the risk factor δ to different values. The findings presented in the Table 4, 5, 8, and 9 indicate how the earliest start times (EST)

Table 10. Activity Duration with respect to different risk factor.

Risk factor (δ)	Technique 1	Technique 2
0	21.08	20.93
0.1	19.6707	19.205
0.2	18.2613	17.48
0.3	16.852	15.755
0.4	15.4427	14.03
0.5	14.0333	12.305
0.6	12.624	10.58
0.7	11.2147	8.855
0.8	9.8053	7.13
0.9	8.426	5.405
1	6.9867	3.68

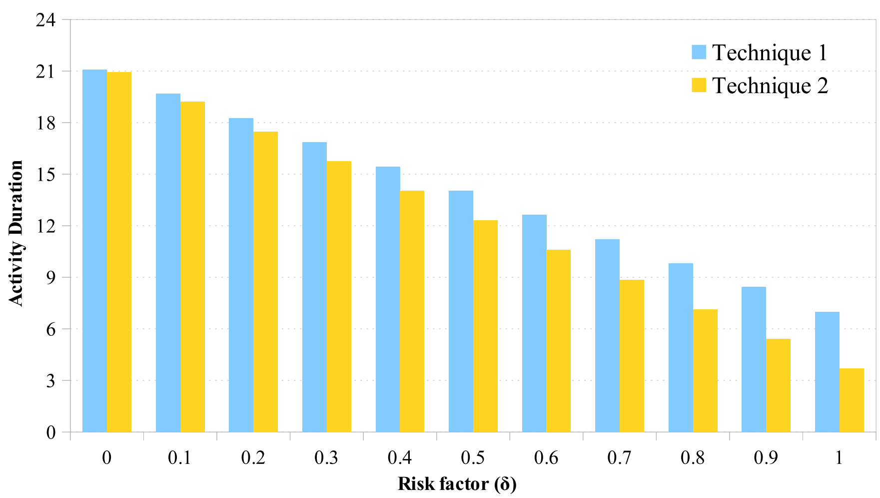

Figure 8. Comparision of Activity duration using Technique 1 and Technique 2.

and latest finish times (LFT) are affected for the various activities by making δ larger. When the risk factor is increased, the values of the EST and LFT decrease, indicating that there is even more uncertainty and hesitation in project scheduling. This suggests the need for selecting an appropriate risk factor δ according to the project situation and risk preference. In general, the findings outline that the methods proposed in this IFCPM study is in fact adaptable to varying degrees of uncertainty, and will help to make decisions in a fuzzy project context.

The suggested IFCPM approachs has advantages, but it is also limited in several respects. Among other issues, it used TIFNs for the representation of activity times, which may not capture uncertainty as effectively as trapezoidal fuzzy numbers (and other fuzzy numbers like Gaussian) in complicated projects. The method also does not consider available resources, which can generate issues in scheduling, and its computationally intensive calculations have scalability limitations when dealing with larger projects, as the number of activities increases as well as the computations. Future rounds of research could consider the integration of alternate fuzzy number representations, utilization of MCDM components in scheduling to help improve decision making, and consideration of integrating IFCPM with AI/ML to augment adaptive scheduling in project scenarios. The IFCPM could also benefit from useful applications that addresses constrained resources through heuristics or metaheuristic optimizations in project execution to advantage of the method for enhanced project performance, efficiency, and effectiveness in practice.

Author Contribution

S. Pati: methodology, software, and editing. K. K. Mohanta: conceptualization, writing and editing. All authors have read and agreed to the published version of the manuscript.

Funding

The authors declare that no external funding or support was received for the research presented in this paper.

Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflicts of Interest

The authors declare that there is no conflict of interest concerning the reported research findings.

References

[1] L. A. Zadeh, "Fuzzy sets," *Information and control*, vol. 8, no. 3, pp. 338–353, 1965.
[2] G. Klir and B. Yuan, *Fuzzy sets and fuzzy logic*, vol. 4. Prentice hall New Jersey, 1995.
[3] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.
[4] R. R. Yager, "Uncertainty modeling using fuzzy measures," *Knowledge-Based Systems*, vol. 92, pp. 1–8, 2016.
[5] Z. Xu, "Intuitionistic preference relations and their application in group decision making," *Information sciences*, vol. 177, no. 11, pp. 2363–2379, 2007.
[6] K. K. Mohanta and D. S. Sharanappa, "A novel technique for solving intuitionistic fuzzy dea model: an application in indian agriculture sector," *Management System Engineering*, vol. 2, no. 1, p. 12, 2023.
[7] V. Chaubey, D. S. Sharanappa, K. K. Mohanta, and R. Verma, "A malmquist fuzzy data envelopment analysis model for performance evaluation of rural healthcare systems," *Healthcare Analytics*, vol. 6, p. 100357, 2024.
[8] K. K. Mohanta, D. S. Sharanappa, D. Dabke, L. N. Mishra, and V. N. Mishra, "Data envelopment analysis on the context of spherical fuzzy inputs and outputs," *European journal of pure and applied mathematics*, vol. 15, no. 3, pp. 1158–1179, 2022.
[9] K. U. Madhuri, S. Siresha, and N. R. Shankar, "A new approach for solving fuzzy critical path problem using ll fuzzy numbers," *Applied Mathematical Sciences*, vol. 6, no. 27, pp. 1303–1324, 2012.
[10] N. R. Shankar, V. Sireesha, K. S. Rao, and N. Vani, "Fuzzy critical path method based on metric distance ranking of fuzzy numbers," *International Journal of Mathematical Analysis*, vol. 4, no. 20, pp. 995–1006, 2010.
[11] H. A. E.-W. Khalifa, M. G. Alharbi, and P. Kumar, "On determining the critical path of activity network with normalized heptagonal fuzzy data," *Wireless Communications and Mobile Computing*, vol. 2021, no. 1, p. 6699403, 2021.
[12] P. P. Bushan Rao and N. Ravi Shankar, "Fuzzy critical path analysis based on centroid of centroids of fuzzy numbers and new subtraction method," *International Journal of Mathematics in operational research*, vol. 5, no. 2, pp. 205–224, 2013.
[13] A. H. Ganesh and A. H. Shobana, "On optimizing interval data based critical path problem for the analysis of various fuzzy quantities using fuzzification and centroid based defuzzification techniques," *Mat. S (1)*, pp. 319–330, 2020.
[14] A. Kumar and P. Kaur, "A new approach for fuzzy critical path analysis," *International Journal of Mathematics in Operational Research*, vol. 3, no. 3, pp. 341–357, 2011.
[15] S. Narayanamoorthy and S. Maheswari, "The intelligence of octagonal fuzzy number to determine the fuzzy critical path: A new ranking method," *Scientific Programming*, vol. 2016, no. 1, p. 6158208, 2016.
[16] M. Gül, A. F. Güneri, and G. Güneş, "Project management in healthcare: A case study for patient flow evaluation in an emergency room using fuzzy cpm and fuzzy pert," *Sigma*, vol. 8, no. 1, pp. 41–51, 2017.
[17] D. Castro-Lacouture, G. A. Süer, J. Gonzalez-Joaqui, and J. Yates, "Construction project scheduling with time, cost, and material restrictions using fuzzy mathematical models and critical path method," *Journal of Construction Engineering and Management*, vol. 135, no. 10, pp. 1096–1104, 2009.
[18] M. Oladeinde and D. Itsisor, "Application of fuzzy theory to project scheduling with critical path method," *Journal of Applied Sciences and Environmental Management*, vol. 17, no. 1, pp. 161–166, 2013.
[19] D. Sen, D. Roy, and S. Dey, "Critical path method in the network analysis with parametric fuzzy activity," *a a*, vol. 1, no. 2, p. 3, 2019.
[20] M. Kiruthiga and T. Hemalatha, "Intuitionistic fuzzy technique to find the critical path," *International Journal of Statistics and Applied Mathematics, 4 (3)*, pp. 39–42, 2019.
[21] N. Rameshan and D. S. Dinagar, "A method for finding critical path with symmetric octagonal intuitionistic fuzzy numbers," *Advances in Mathematics: Scientific Journal*, vol. 9, no. 11, pp. 9273–9286, 2020.
[22] R. S. Porchelvi and G. Sudha, "Critical path analysis in a project network using ranking method in intuitionistic fuzzy environment," *Int. J. Adv. Res*, vol. 3, pp. 14–20, 2015.
[23] G. Sudha, R. Porchelvi, and K. Gnanaselvi, "Solving critical path problem using triangular intuitionistic fuzzy number," *International Journal of Fuzzy Mathematical Archive*, vol. 14, no. 1, pp. 1–8, 2017.

[24] T.-C. Han, C.-C. Chung, and G.-S. Liang, "Application of fuzzy critical path method to airports cargo ground operation systems," *Journal of Marine Science and Technology*, vol. 14, no. 3, p. 2, 2006.
[25] T. Yogashanthi, S. Sathish, and K. Ganesan, "A study on intuitionistic fuzzy critical path problems through centroid based ranking method," *Math. Stat.*, vol. 10, no. 6, pp. 1326–1333, 2022.
[26] P. Jayagowri and G. Geetharamani, "A critical path problem using intuitionistic triangular fuzzy number," *International Journal of Engineering Research and Applications*, vol. 4, no. 5, pp. 55–60, 2014.
[27] R. Shaik and N. R. Shankar, "Ranking method and graphical method to find the critical path of a fuzzy project network with trapezoidal intuitionistic fuzzy numbers as activities," *Tuijin Jishu/Journal of Propulsion Technology*, vol. 45, no. 1, p. 2024.
[28] P. Kaur and A. Kumar, "A new ranking approach and its application to solving fuzzy critical path problems," *South African Journal of Industrial Engineering*, vol. 23, no. 3, pp. 91–102, 2012.